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Summary of the doctoral dissertation in English

Title of the doctoral dissertation:

Proof systems for some many-valued and modal logics

The aim of this thesis is to explore proof systems for various modal, many-valued, and modal many-valued logics. We consider two kinds of proof systems: sequent calculi (and their generalisations: hypersequent and nested sequent calculi) and natural deduction systems. Sequent calculi are known to be a good theoretical tool for the investigation of proofs; they visually and clearly represent the structure of the proofs and allow the distinction between logical and structural rules to distinguish the properties of the logical connectives and the properties of the consequence relation. The advantage of natural deduction systems is their similarity to the process of natural human reasoning.

One of the central theorems of proof theory is the cut elimination theorem, or the cut admissibility theorem. The cut elimination theorem says that if in a sequent calculus with a cut rule as a primitive we are able to prove some sequent, then we are able to prove the same sequent in its cut-free version. The cut admissibility theorem says that if in a cut-free sequent calculus we have proofs of the premisses of a cut rule, then without using the cut rule itself, we are able to get its conclusion. As a consequence of the cut elimination/admissibility theorem, it is often possible to reach the subformula property (or some of its restricted forms), interpolation, decidability, and other important results.

In the case of natural deduction systems, there is an analogue of the cut elimination/admissibility theorem: the normalisation theorem. It says that in the proof there are no maximal formulas (the formulas that are conclusions of introduction rules and major premisses of elimination rules) and no maximal segments (i.e., sequences of maximal formulas). In our case, we will use a slightly different form of normalisation since we are going to deal with general elimination and general introduction rules (such rules better suit

our aims), so for us, a maximal formula is the one that is a major premiss of a general elimination rule and a major assumption of a general introduction rule. Anyway, the normalisation theorem helps establish the subformula property.

During our investigation, we will try to provide for any logic in question both a sequent calculus (or rather, a hypersequent or a nested sequent calculus) and a natural deduction system. We will prove the cut admissibility theorem for all the sequent calculi in question and do that by two methods: a semantic one (as a consequence of a completeness theorem proven by a Hintikka-style argument) and a syntactic constructive one. The normalisation theorem will be proved by the syntactic constructive method. As a consequence, we establish the subformula property.

As we have already said, we explore two different fields of non-classical logics: modal and many-valued ones, including their mixture, modal many-valued logics, and their algebraic generalisation, modal multilattice logics. In general, logical systems can be divided into two groups: tabular and non-tabular ones. The former have finitely-valued semantics. The latter require infinitely-valued semantics or other types of semantics: Kripke, algebraic, topological, etc. Since three- and four-valued logics are the most popular and remarkable representatives of finitely-valued logics, we are going to focus our attention on them. There are lots of various types of non-tabular logics: modal, intuitionistic, linear, temporal, epistemic, doxastic, dynamic, relevant, fuzzy, etc. Since most of them have Kripke semantics, which was originally developed for modal logics, we think that it is reasonable to focus on modal logics. As for modal many-valued logics, they are the bridge between these two fields. We provide several proof-theoretic results of the sort described above for several such logics. In many cases it requires significant changes in the standard methods.